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Novel communication scheme based on chaotic Rössler circuits

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Abstract: We present a novel synchronization scheme for secure communication with two chaotic unidirectionally coupled Rössler circuits. The circuits are synchronized via one of the variables, while a signal is transmitted through another variable. We show that this scheme allows more stable communications. The system dynamics is studied numerically and experimentally in a wide range of a control parameter. The possibility of secure communications with an audio signal is demonstrated.

1. Introduction

In recent years chaos theory has attracted much interest in both the academic area and engineering study. Today the potential of chaos theory is recognized in the worldwide with research groups actively working on this topic [1].

One of the great achievements of the chaos theory is the application in secure communications. The chaos communication fundament is the synchronization of two chaotic systems under suitable conditions if one of the systems is driven by the other. Since Pecora and Carrol [2] have demonstrated that chaotic systems can be synchronized, the research in synchronization of couple chaotic circuits is carried out intensively and some interesting applications such as communications with chaos have come out of that research.

In this work we use a simple electronic system to develop a novel scheme for chaos secure communication with two coupled Rössler circuits. First, we analyze separately each oscillator to study their dynamic behaviour when a parameter of control is changed, and then we investigate the synchronization effect in the coupled circuits. Bifurcation diagrams of the output voltage are constructed using a resistance as a control parameter. While using two channels, we may send an information signal via one of the channels and recover the signal via another channel. We will show that this scheme can improve synchronization in a system with coexisting attractors. Finally secure audio communications with chaos is demonstrated experimentally using the novel communication scheme.

2. Experimental setup

We use the electronic circuits of a Rössler type shown in figure 1. The master circuit can be described by the following equations [3]:

$$\frac{dx}{dt} = -\alpha(\Gamma x + \beta y + \lambda z), \tag{1}$$

$$\frac{dy}{dt} = -\alpha(-x - \gamma y + 0.02 z), \tag{2}$$

$$\frac{dz}{dt} = -\alpha(-g(x) + z), \tag{3}$$

$$g(x) = \begin{cases} 0, & x \le 3 \\ \mu(x-3), & x > 3 \end{cases}, \tag{4}$$

where $\alpha = 10^4\, s^{-1}$, $\Gamma = 0.05$, $\beta = 0.5$, $\lambda = 1.0$, $\mu = 15$, $\gamma = R/R_c$, R = 10 kΩ, and R_c is a control parameter which varied between 1 kΩ and 200 kΩ.

The piecewise linear function $g(x)$ is determined by the diode in the operational amplifier A4. The amplifier is switched on when the voltage X exceeds 3V. The master and slave circuits are identical. When the drive-out signal Y of the master circuit is the drive-in signal of the slave circuit, every change in any parameters of the first circuit affects the second circuit.

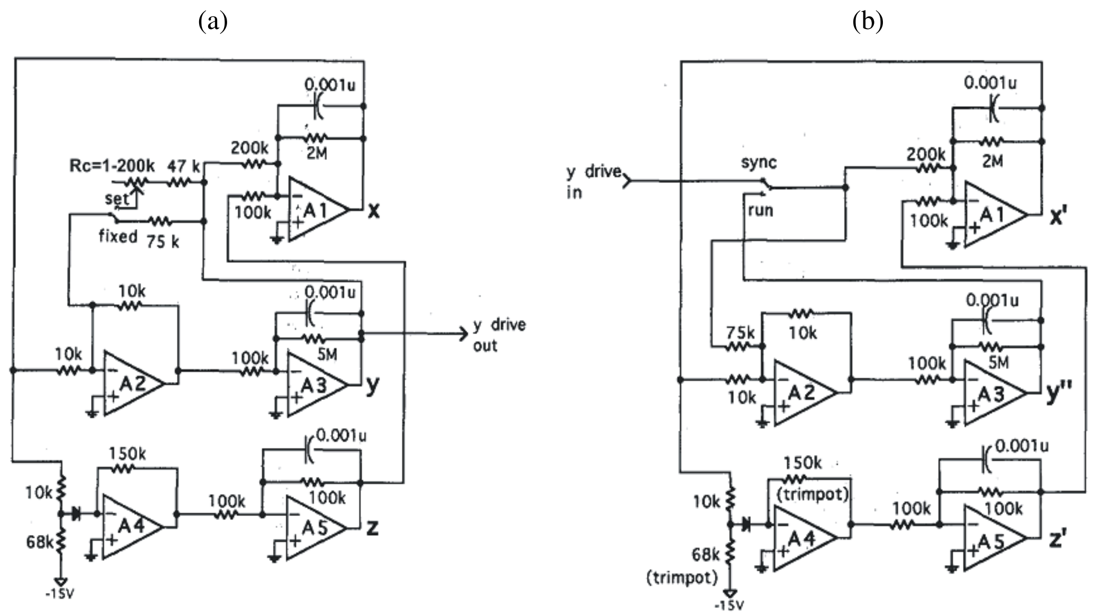
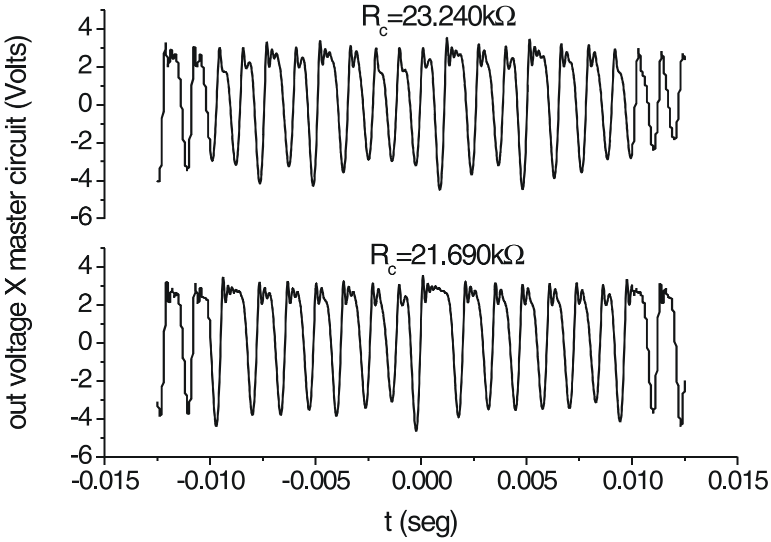


Figure 1. Electronic schemes for (a) master and (b) slave circuits.

The output voltages X, Y and Z are registered with an oscilloscope (Tektronics, series 2000). The experimental time series and phase space of the isolated master circuit are shown in the figure 2. The system displays a homoclinic behavior in the range 22.6 kΩ < R_c < 30 kΩ.

(a)



(b)

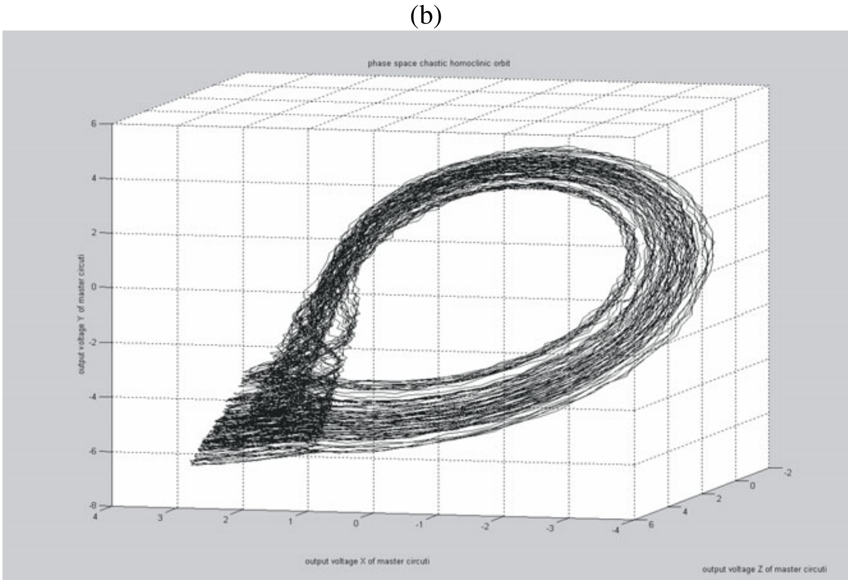


Figure 2. (a) Time series of voltage *X*, (b) chaotic homoclinic trajectory in phase space (*X,Y,Z*).

A homoclinic chaotic orbit has the property that its phase trajectory fluctuates near a critical point in phase space. Figure 2(b) displays the chaotic motion of the homoclinic orbit which is characterized by large fluctuations in return times associated with a high sensitivity of the trajectory when the chaotic trajectory approaches the critical point. For $30 < R_c <$ to 86.6 kΩ the dynamics inhibits typical Rössler chaos, which the time series are shown in figure 3. In the interval 86.6 kΩ $< R_c <$ 107 kΩ the dynamic of the master circuit is periodic and for 107 kΩ $< R_c <$ 200 kΩ the behavior is a steady state.

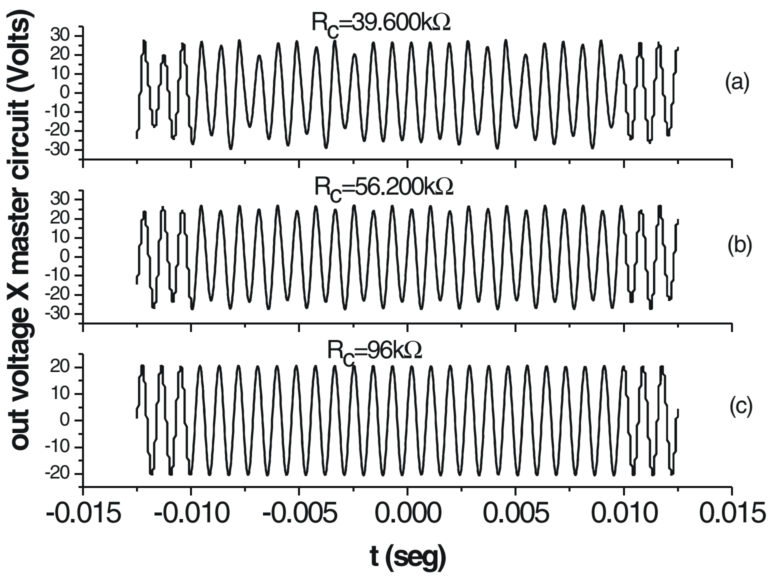


Figure 3. Time series of X for different values of R_c showing (a, b) Rössler chaos and (c) period-1 regime.

3. Numerical simulations

Mathematically, the Rössler circuit can be described by the system of equations (1)-(4). The control parameter is the resistance R_c which varies from 1 kΩ to 200 kΩ. The time series and the trajectories in the phase space are shown in figure 4. For R_c from 28kΩ to 32 kΩ, the solution displays homoclinic orbit behavior, for R_c from 32 kΩ to 96 kΩ the attractors are chaotic of a Rössler type, and for R_c from 96 kΩ to 140 kΩ the dynamics is periodic.

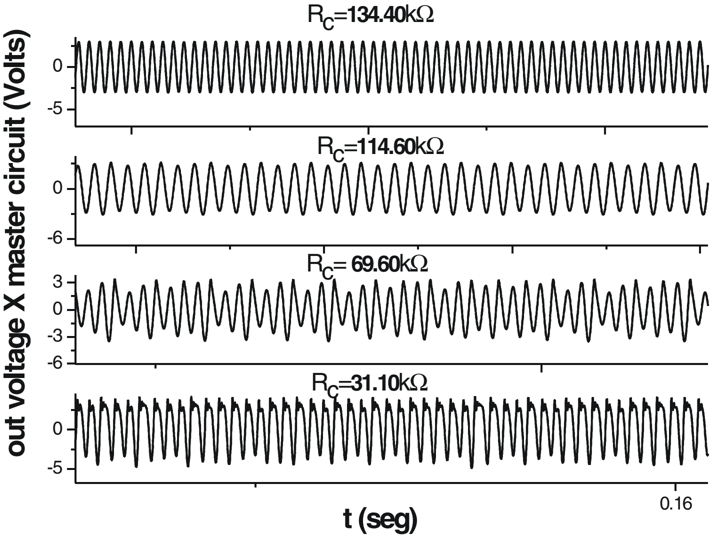
The difference between the experimentally output voltage X of the master circuit and the numerical solutions X of the system of equations (1)–(4) is that there are not numerical solutions for inferior values of R_c = 28 kΩ, whereas experimentally we find the response up to 22.6 kΩ. The reason for this difference is that some elements of the circuits have tolerances in operation conditions (when the circuit is switched on) from its stationary values (when the circuit is switched off). For values of $R_c <$ 20.6 kΩ in experiments and for $R_c <$ 28 kΩ in the simulations the system has no stable solutions.

The results of the numerical simulations are in a good agreement with the experimental results: depending on parameter R_c the master circuits displays homoclinic orbits, Rössler chaos and periodic orbits. This complex dynamic behavior is clearly seen in the bifurcation diagrams in figure 5. In experiments the control parameter is varied by the potentiometer R_c. In both diagrams we observe regions of homoclinic orbits for small values of R_c, and Rössler chaos with period windows for higher values. For very large R_c only periodic orbits are observed.

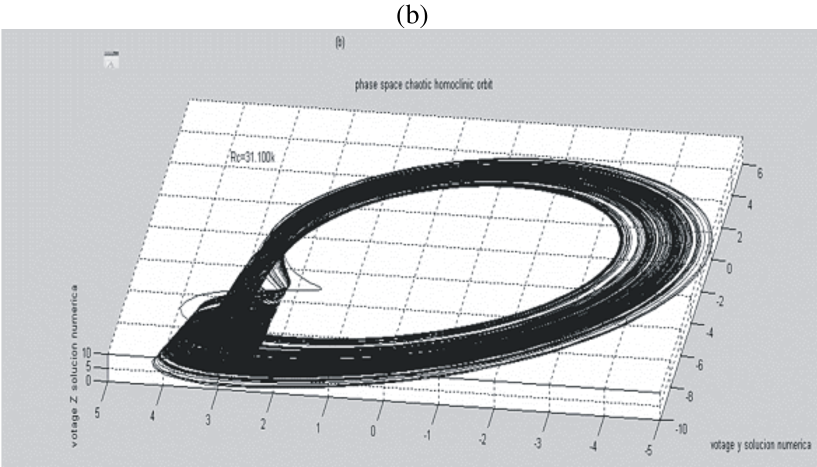
4. Synchronization of two coupled chaotic Rössler circuits

Synchronization of two coupled chaotic Rössler circuits is studied by connecting the output voltage Y of the master circuit to the input Y of the slave circuit. By varying the control parameter R_c in the master circuit we can find synchronous regimes between the master and slave circuits. In figure 6(a) we show the temporal series of the output voltages of the master and slave circuits, Y and Y', when they are not coupled. The trajectory in the phase space (Y,Y') is shown in figure 6(b). It is clearly seen that the circuits are not synchronized.

(a)



(b)



(c)

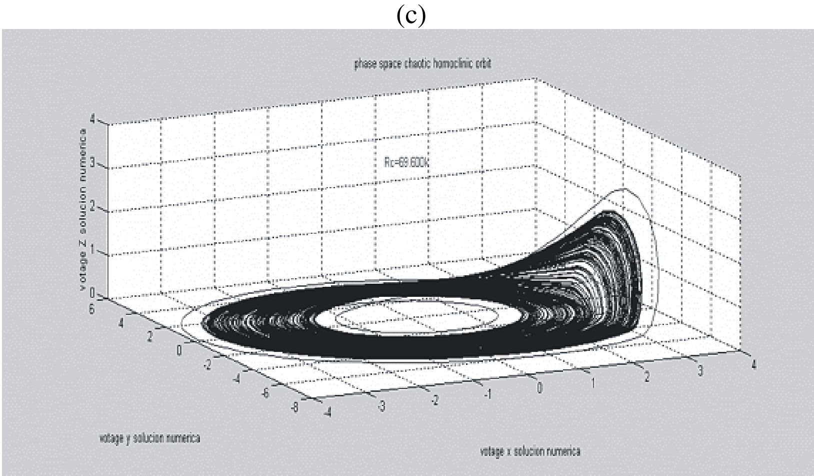


Figure 4. (a) Numerical time series of equations (1)-(4) and (b)–(c) phase space trajectories showing chaotic homoclinic Rössler attractors.

(a) (b)

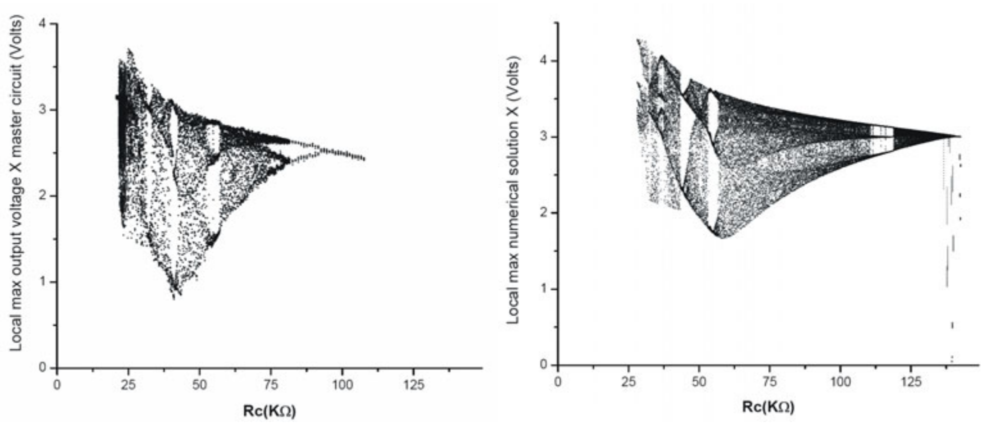
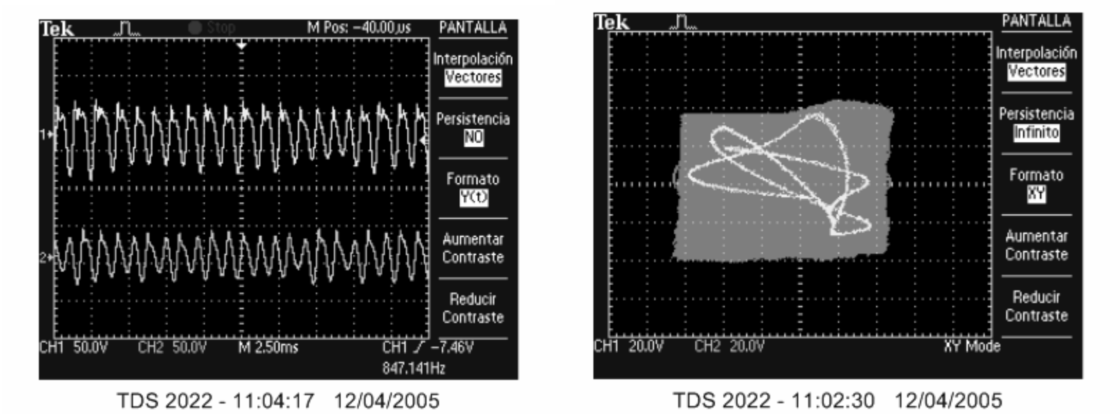


Figure 5. (a) Experimental and (b) Numerical bifurcations diagrams of *X* of master circuits.

(a) (b)



(c) (d)

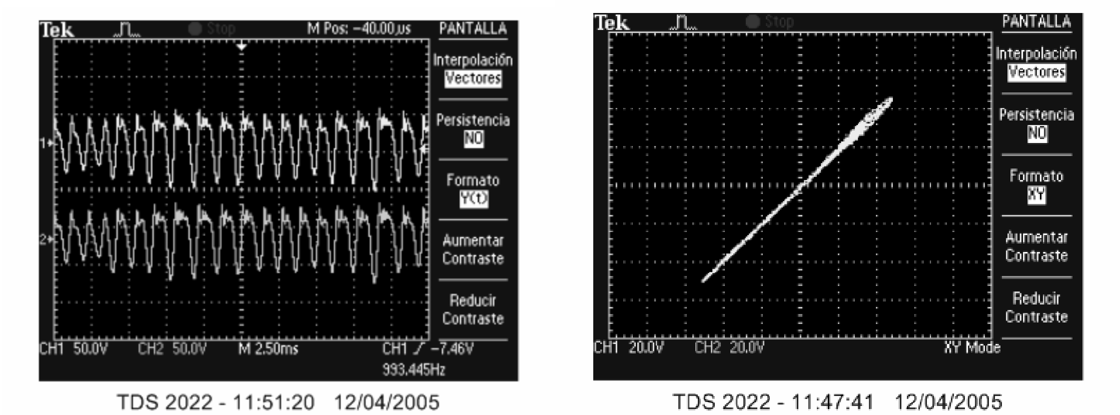
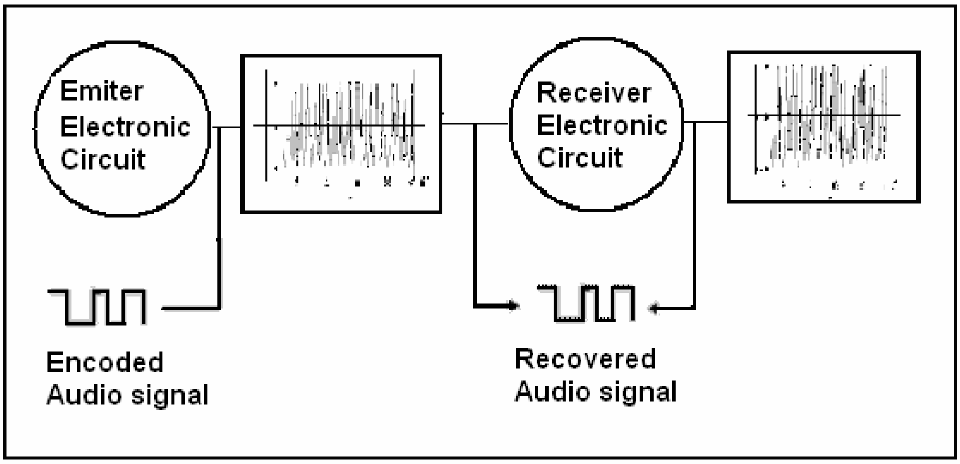


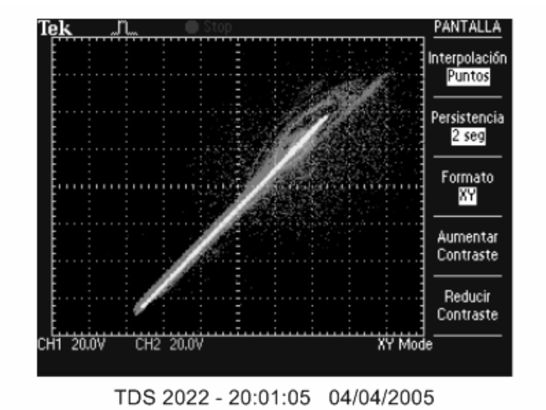
Figure 6. (a) Time series and (b) phase space trajectories Y_M and Y_S without coupling, and (c) and (d) the same with coupling.

When the circuits are coupled, i.e. the output variable *Y* of the master circuit is the input variable *Y'* of the slave circuit, the circuits are completely synchronized.

(a)



(b) (c)



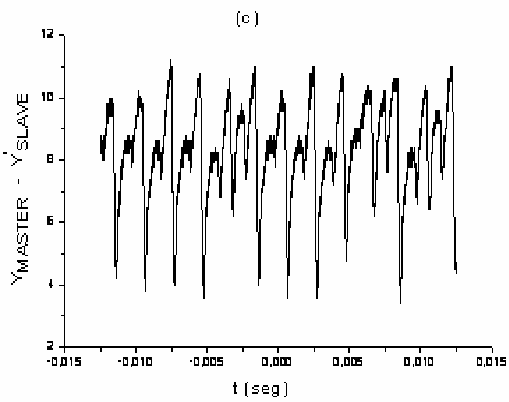


Figure 7. (a) Classical communication scheme, (b) intermittent synchronization between master and slave circuits, and (c) synchronization error.

5. Secure communication with chaotic circuits

In a traditional chaos communication system a small information signal *s*(*t*) is added to a chaotic signal *Y*(*t*) and it is transmitted, and at a receiver, the transmitted signal *s*(*t*)+*Y*(*t*) is used to synchronize the identical chaotic system. The synchronous chaotic signal at the receiver is then used to recover the information signal from the transmitter. Figure 7(a) illustrates this communication scheme [4].

In implementation of the traditional scheme to our circuits yields incomplete synchronization shown in figure 7(b). When we add a small rectangular information signal *s*(*t*) to the output voltage of the master circuit *Y*(*t*), the sum signal *Y*(*t*)+*s*(*t*) is used to couple the slave circuit. We observe that the output voltage of the slave circuit *Y''*(*t*) is synchronized intermittently with the input signal, as show in figure 7(b). This intermittent synchronous regimen does not allow us to recover the information signal.

(a)

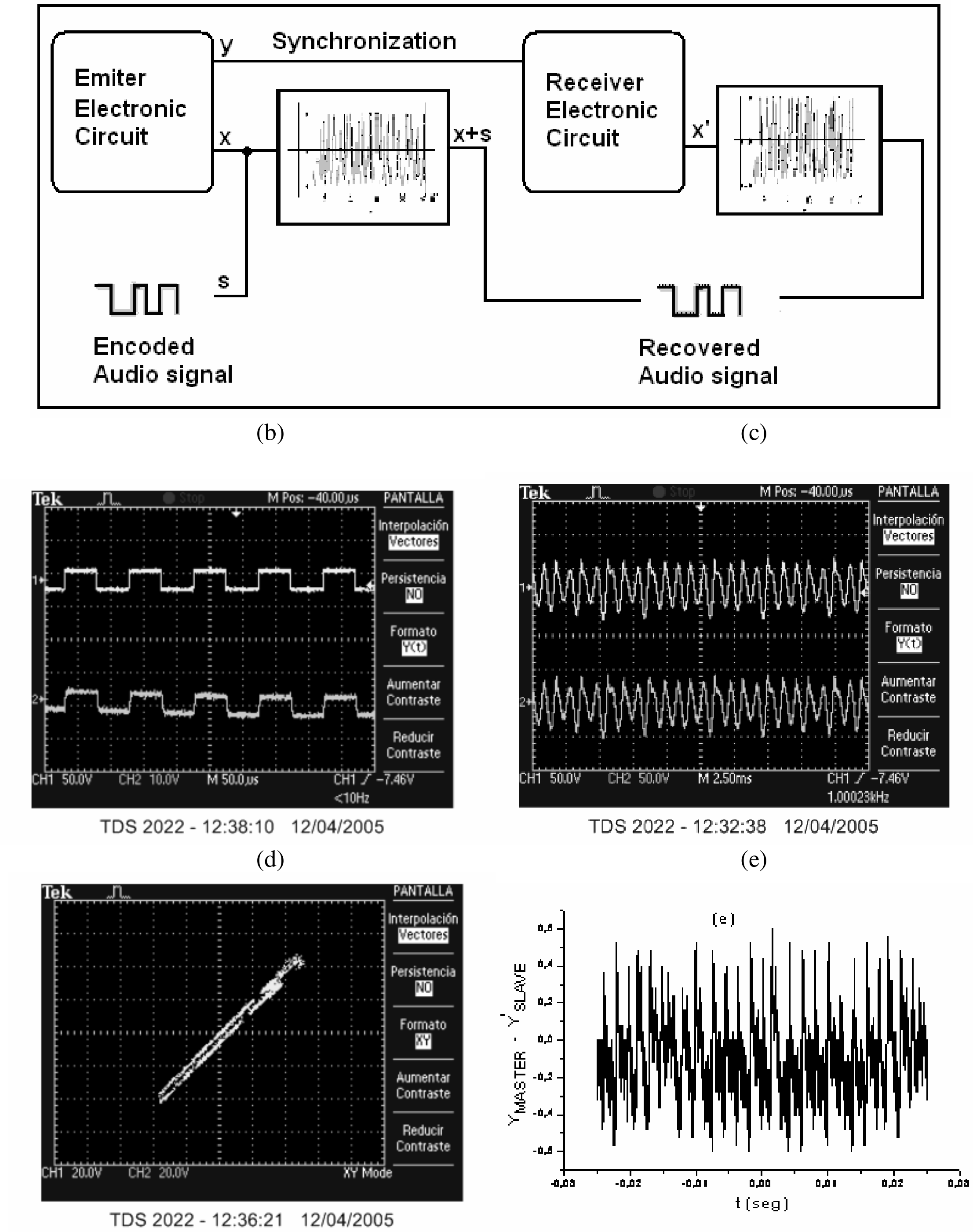


Figure 8. (a) Novel synchronization scheme for secure communication with chaotic Rössler circuits, (b) information signal $s(t)$, and (c) chaotic output voltage $X(t)$ (upper trace) and $X(t)+s(t)$ (lower trace), (d) complete synchronization of X variables, (e) synchronization error.

The origin of this intermittency is the coexistence of attractors in the coupled system [5]. The information signal provokes jumps between the attractors resulting in intermittency. In order to avoid this inconvenience we can take an advantage that the master circuit has three variables *X, Y,* and *Z* which can be used for synchronization and recovering the information signal. The sensitivity of the system to a change in a variable may be different for different variables. For our system the variable Y is more sensitive to a small perturbation than X. Therefore the variable Y is more convenient for synchronization, while the variable X is used for signal transmission and encoding. The novel communication scheme is illustrated in Figure 8(a).

The information signal $s(t)$ and the chaotic output $X(t)$ without signal and with signal, $X(t)+s(t)$ are shown in figures 8(b) and 8(c), respectively. In figure 8(d) we showed synchronization between $X(t)+s(t)$. We can see that the synchronization error shown in figure 8(c) is smaller than the error for the traditional scheme in figure 7(a).

6. Conclusion

We propose a novel communication scheme for secure communications based on synchronization of chaotic systems. The scheme implies the use of two system variables, the one serves for chaos synchronizations and the other is used for signal transmission and recovering. We show that the synchronization error for the novel scheme is smaller than that for the traditional scheme.

The main advantage of the new communication scheme over the traditional one is that when we use two channels, sending the information signal via one channel and synchronizing master and slave circuits via another channel, we can obtain higher stability in the recovered signal for systems with coexisting attractors.

The intermittent synchronization regime can be used for further improvement of communication security if a proper control of intermittency is organized [6].

7. Acknowledgment

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References

[1] Kennedy M P, Rovatti R and Setti G 2000 *Chaotic Electronics in Telecommunications* (CRC Press, USA)

[2] Carrol T and Pecora L 1995 *Nonlinear Dynamics in Circuits* (World Scientific Publishing, Singapore)

[3] Carroll T L 1995 *Am J Phys* **63** 377

[4] Cuomo K M, Oppenheim A V and Strogatz S H 1993 *IEEE Trans Circuits Syst II* **40** (10) 626

[5] Pisarchik A N and Jaimes-Reátegui R 2005 *Phys Lett A* **338** 141

[6] Reategui R J and Pisarchik A N 2004 *Phys Rev E* **69** 067203